

March 10, 1887.

Professor STOKES, D.C.L., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

- I. "Note on Induction Coils or 'Transformers.'" By JOHN HOPKINSON, M.A., D.Sc., F.R.S. Received February 17, 1887.

The transformers considered are those having a continuous iron magnetic circuit of uniform section.*

Let A be area of section of the core,
 m and n the number of convolutions of the primary and secondary coils respectively,
 R , r , and ρ their resistances, ρ being the resistance of the secondary external to the transformer,
 x and y currents in the two coils,
 α induction per square centimetre,
 α the magnetic force,
 l the length of the magnetic circuit,
 $E = B \sin 2\pi(t/T)$, the difference of potentials between the extremities of the primary,
 T being the periodic time.

We have

$$(1.) 4\pi(mx + ny) = l\alpha;$$

$$(2.) E = Rx - mA\dot{\alpha};$$

$$(3.) 0 = (r + \rho)y - nA\dot{\alpha};$$

from (2) and (3),

* For a discussion of transformers in which there is a considerable gap in the magnetic circuit, see Ferraris, 'Torino, Accad. Sci. Mem.,' vol. 37, 1885; Hopkinson, "On the Theory of Alternate Currents," 'Telegr. Engin. Journ.,' vol. 13, 1884, p. 496.

$$(4.) nE = nRx - m(r + \rho)y;$$

substituting from (1),

$$(5.) x\{n^2R + m^2(r + \rho)\} = n^2E + (l\alpha/4\pi)m(r + \rho),$$

$$(6.) y\{n^2R + m^2(r + \rho)\} = -nmE + (l\alpha/4\pi)nR;$$

$$(7.) A\dot{a} = -\frac{(r + \rho)mE}{n^2R + m^2(r + \rho)} + \frac{l\alpha R(r + \rho)}{4\pi\{n^2R + m^2(r + \rho)\}}.$$

We may now advantageously make a first approximation, neglect $l\alpha$ in comparison with $4\pi m\alpha$, that is, assume the permeability to be very large, we have

$$(8.) A\dot{a} = -\frac{(r + \rho)mB \sin(2\pi t/T)}{n^2R + m^2(r + \rho)};$$

$$(9.) Aa = \frac{(r + \rho)mB \cos(2\pi t/T)}{\{n^2R + m^2(r + \rho)\} \cdot 2\pi t/T}.$$

For practical purposes these equations are really sufficient.

We see firstly that the transformer transforms the potential in the ratio n/m , and adds to the external resistance of the secondary circuit ρ a resistance $(n^2R/m^2) + r$. This at once gives us the variation of potential caused by varying the number of lamps used. The phase of the secondary current is exactly opposite to that of the primary.

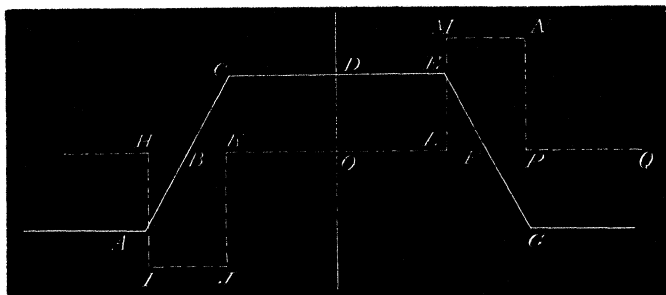
In designing a transformer it is particularly necessary to take note of equation (9), for the assumption is that a is limited so that $l\alpha$ may be neglected. The greatest value of a is $B/\{(2\pi/T)mA\}$, and this must not exceed a chosen value. We observe that B varies as the number of reversals of the primary current per unit of time.

But this first approximation, though enough for practical work, gives no account of what happens when transformers are worked so that the iron is nearly saturated, or how energy is wasted in the iron core by the continual reversal of its magnetism. The amount of such waste is easily estimated from Ewing's results when the extreme value of α is known, but it is more instructive to proceed to a second approximation, and see how the magnetic properties of the iron affect the value and phase of x and y . We shall as a second approximation substitute in equations (5) (6) (7) values of α deduced from the value of a furnished by the first approximation in equation (9).

In the accompanying diagram Ox represents α , Oy represents a , and Oz the time t .

The curves ABCD represent the relations of a and α . EFG the induction a as a function of the time, and HIK the deduced relation between α and t . We may substitute the values of α obtained

FIG. 3.



resulting relations of potential in the secondary and the time will be indicated by the dotted line HIJKOLMNPQ. The mean square observed will be proportional to $ML \cdot \sqrt{LP}$; but $ML \cdot LP$ is proportional to EL , hence the potential observed will vary inversely as \sqrt{LP} , even though the maximum induction remain constant. If then the maximum induction be deduced on the assumption that the induction is a simple harmonic function of the time, results may readily be obtained vastly in excess of the truth.

II. "Note on the Theory of the Alternate Current Dynamo."

By JOHN HOPKINSON, M.A., D.Sc., F.R.S. Received February 17, 1887.

According to the accepted theory of the alternate current dynamo, the equation of electric current in the armature is $\gamma \dot{y} + Ry =$ periodic function of t , where γ is a constant coefficient of self-induction. This equation is not strictly true, inasmuch as γ is not in general constant,* but it is a most useful approximation. My present purpose is to indicate how the values of γ and of the periodic function representing the electromotive force can be calculated in a machine of given configuration.

To fix ideas, we will suppose the machine considered to have its magnet cores arranged parallel to the axis of rotation, that the cores are of uniform section, also that the armature bobbins have iron cores, so that we regard all the lines of induction as passing either through an armature coil, or else between adjacent poles entirely outside the armature. The sketch shows a development of the machine considered. The iron is supposed to be so arranged that the currents

* "On the Theory of Alternating Currents," 'Telegr. Engin. Journ.,' vol. 13, 1884, p. 496.

FIG. 1.

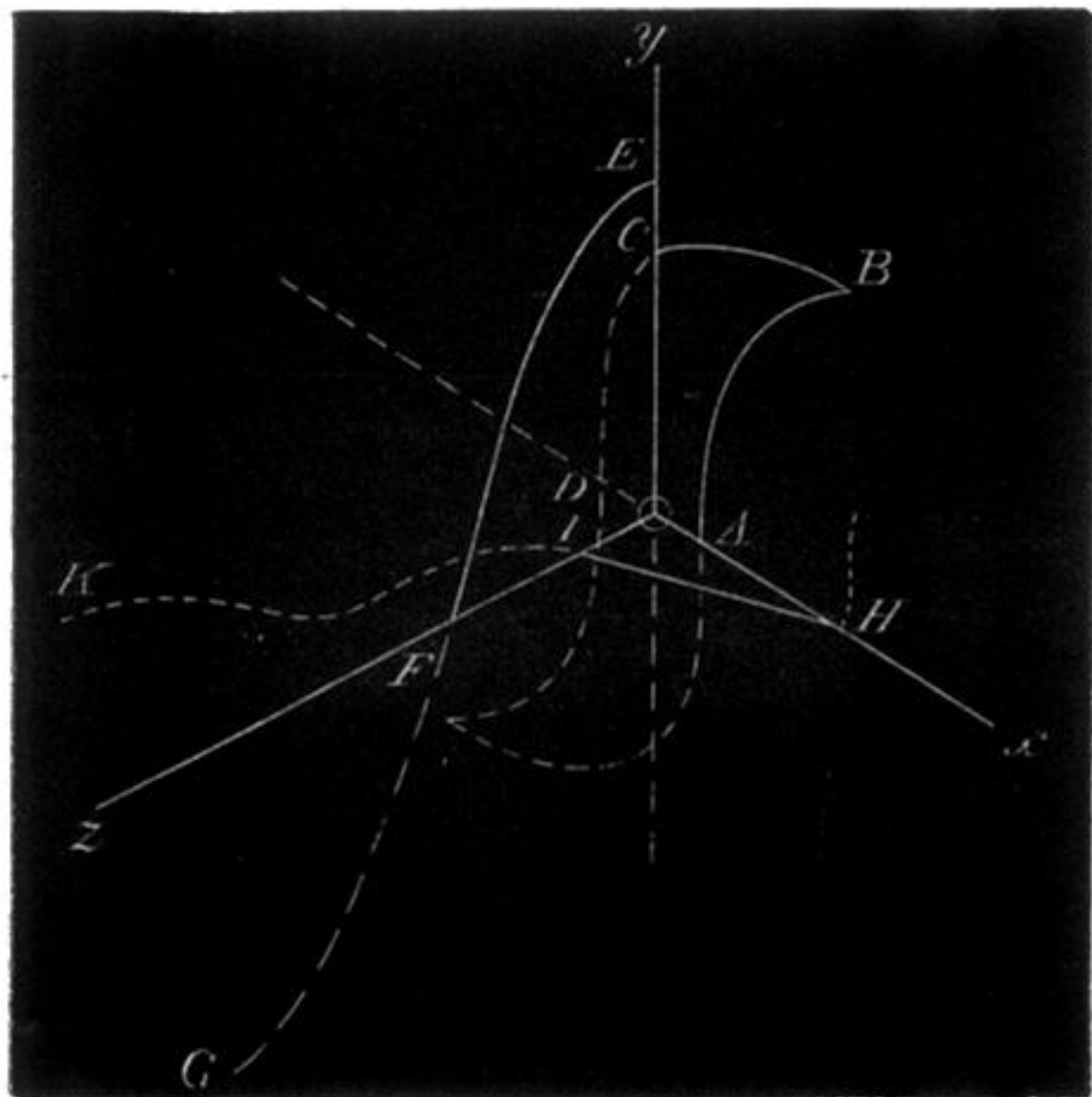


FIG. 2.

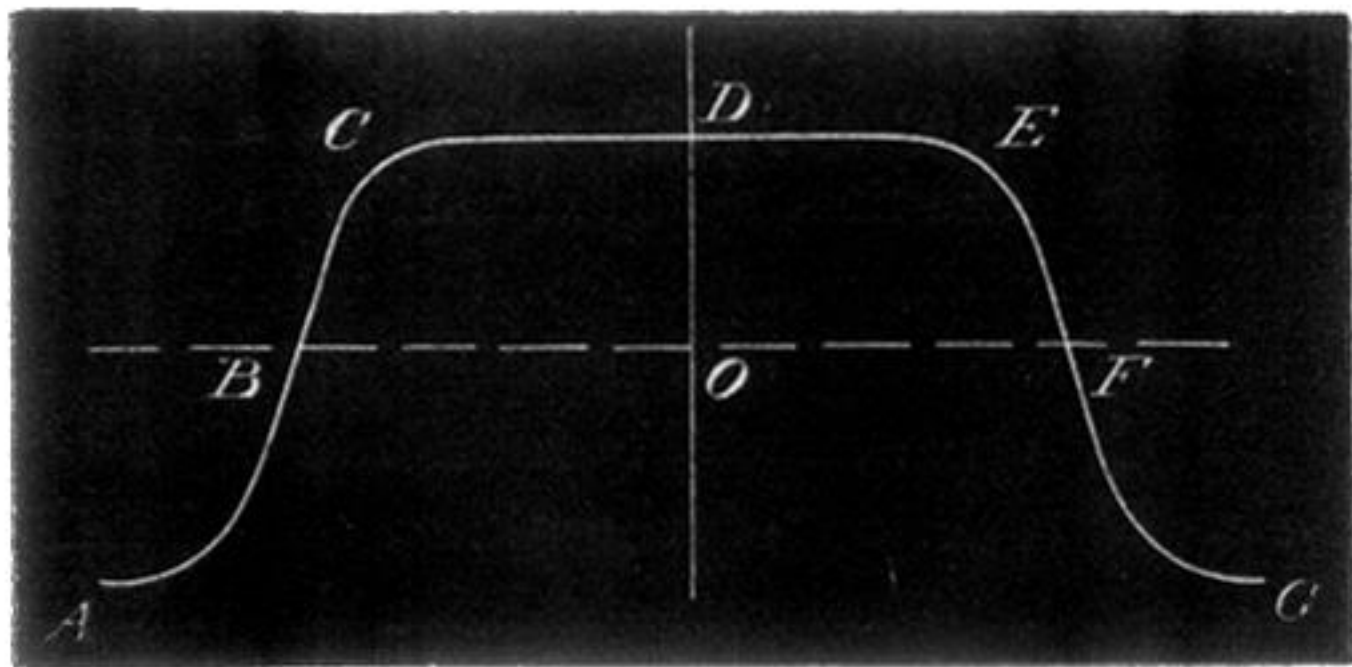


FIG. 3.

